Table 3 Comparison of objective function, design variable, and constraint values

Parameter	Initial	Optimal
Objective function f/W^2	0.8743	0.8509
Design variable		
a/W	0.400	0.336
b/W	0.400	0.565
Constraint σ_e/σ_a		
g_1	0.8886	0.9940
g_2	0.3022	0.6452
g_3	1.0695	1.0086

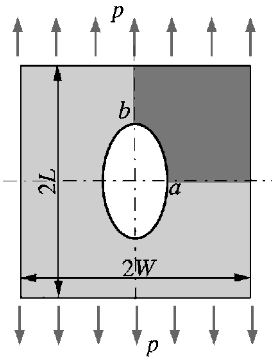

Fig. 1 Plate with an elliptical hole under uniaxial tension.

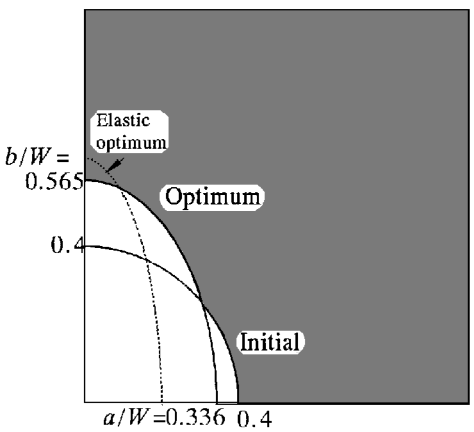

Fig. 2 Initial and optimum shapes of elliptical holes.

subdivided into 48 8-node quadratic elements and 173 nodes. The finite element mesh in all regions is adjusted adaptively at each optimization step. Eight loading steps of $\Delta p_1 = 3p_0$, $\Delta p_2 = \Delta p_3 = \Delta p_4 = \Delta p_5 = \Delta p_6 = \Delta p_7 = \Delta p_8 = p_0$ are applied incrementally.

The hole shape is interpolated by trigonometric function as $x_1 = a\cos\theta$ and $x_2 = b\sin\theta$, and the major and minor semiaxes a and b are taken as the design variables. The equivalent values of the von Mises stress on the hole boundary are restricted less than the allowable stress $\sigma_a/\sigma_{Y0} = 1.25$, and the stress constraints are imposed at several points on the boundary. To minimize the weight, the direct Taylor series approximation technique is adopted for constructing a quadratic subproblem. The approximated subproblem with move limit is solved by the complementary pivot method. The optimum solution of the design variables, the objective function, and the stress constraints obtained after seven iterations are tabulated in Table 3 in comparison with the initial values. The stress constraints at $\theta = 0$ deg (g_1) and 90 deg (g_3) are active in the optimum shape. Figure 2 shows the initial and optimum shapes of the hole. The optimum hole shape of the elastic design with same allowable stress is also shown as a reference. The optimum hole shape of elastic design ($b/a = 3.10$) is smaller and more slender than that of the plastic design ($b/a = 1.68$).

Concluding Remarks

An exact and direct sensitivity analysis technique of the elastoplastic material governed by bilinear constitutive law, which has the discontinuous sensitivities at the yielding point, has been suggested. The sensitivity analysis technique was applied to the two-dimensional sensitivity analysis and shape optimization of the elastoplastic material. From these numerical results, it was found that 1) the sensitivity analysis technique suggested here can exactly treat the discontinuity caused by the bilinear constitutive law of elastoplastic material, and 2) the sensitivity analysis technique can effectively determine the optimum shape of an elastoplastic two-dimensional body with the aid of the approximation method.

References

[1]Kaneko, I., and Majer, G., "Optimal Design of Plastic Structures Under Displacement Constraints," *Computer Methods in Applied Mechanics and Engineering*, Vol. 27, 1981, pp. 369–391.

[2]Ryu, Y. S., Haririan, M., Wu, C. C., and Arora, J. S., "Structural Design Sensitivity Analysis of Nonlinear Response," *Computers and Structures*, Vol. 21, Nos. 1/2, 1985, pp. 245–255.

[3]Cardoso, J. B., and Arora, J. S., "Variational Method for Design Sensitivity Analysis in Nonlinear Structural Mechanics," *AIAA Journal*, Vol. 26, No. 5, 1988, pp. 595–603.

[4]Arora, J. S., Lee, T. H., and Kumar, V., "Design Sensitivity Analysis of Nonlinear Structures—III: Shape Variation of Viscoplastic Structures," *Structural Optimization: Status and Promise*, edited by M. P. Kamat, AIAA, Washington, DC, 1993, pp. 447–485.

[5]Cheng, G. D., and Song, L., "Computational Aspects of Sensitivity Analysis of Nonlinear Discrete Structures," *Report of Institute of Mechanical Engineering*, No. 34, Aalborg Univ., Denmark, 1991.

[6]Jao, S. Y., and Arora, J. S., "Design Optimization of Non-Linear Structures with Rate-Dependent and Rate-Independent Constitutive Models," *International Journal for Numerical Methods in Engineering*, Vol. 36, No. 16, 1993, pp. 2805–2823.

[7]Ohsaki, M., and Arora, J. S., "Design Sensitivity Analysis of Elastoplastic Structures," *International Journal for Numerical Methods in Engineering*, Vol. 37, No. 2, 1994, pp. 737–762.

[8]Hisada, T., "Basic Formulation for Elastic–Plastic Stochastic Finite Element Method (Sensitivity Analysis of Nonlinear FEM)," *Transactions of Japanese Society of Mechanical Engineers*, Vol. 56, No. 524, 1990, pp. 966–970 (in Japanese).

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Interaction Region of Turbulent Expansion-Corner Flow

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Introduction

THE Prandtl–Meyer solution for the flow around an expansion corner is well known. However, the study of Adamson[1] indicated that the transport properties in the expansion process of a real flow may have strong effects on the laminar flow properties. The velocity and streamline patterns change, and the surface pressure decreases gradually along the streamwise direction. The flow would reach the final equilibrium condition only after some finite distance downstream of the corner. The previous studies[2–6] at Mach 1.76–8.0 indicated the similar trend of the expansion process. Further, Lu and Chung[6] found that the downstream influence (x_D/δ_o) of turbulent flow past expansion corners can be scaled with the hypersonic similarity parameter ($K = M_\infty\alpha$) (Fig. 1). The surface pressure of weak expansions reaches the downstream inviscid value more quickly. However, Narasimha and Sreenivasan[7] mentioned that the interaction region appears to be insensitive to the corner deflection angle for supersonic flow. In this Note, Narasimha and Sreenivasan's statement is corroborated for a low supersonic Mach number in conjunction with the work of Lu and Chung,[6] which allows conclusions to be drawn concerning Mach number effects on

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the extent of the interaction region for turbulent expansion corner flows. In addition, an interaction region based on surface pressure fluctuations is illustrated.

Interaction Region

In the present experiment, the stagnation pressure and temperature are 193 ± 1 kPa and room temperature, respectively. The incoming Mach number is 1.280 ± 0.003, and the unit Reynolds number is 2.57×10^7 per meter. Tests are conducted for the incoming flow past 5-, 10-, and 15-deg expansion corners, where $K = 0.112$, 0.223, and 0.335. For the surface pressure measurements, one row of 19 pressure taps along the centerline of the plate is drilled perpendicularly to the test surface. Kulite Model XCS-093-25A pressure transducers are flush mounted to obtain the dynamic data. The incoming boundary-layer thickness at $x_{le} = 485$ mm is estimated to be 7.0 ± 0.2 mm by the pitot probe measurement, and the normalized velocity profile appears to be full ($n \approx 9$ for the velocity power law). This indicates the turbulent flows at the measurement locations.

The mean surface pressure distributions for the flow around the expansion corners are plotted in Fig. 2. The pressure is plotted in nondimensional form, using the stagnation pressure p_o as reference, against $x^* = x/\delta_o$, where x is the streamwise surface coordinate center at the corner and δ_o is the incoming boundary-layer thickness. The inviscid pressure distributions are also shown for comparison. It can be seen that the mean surface pressure distributions show some upstream influence. The flows accelerate slightly in front of the expansion corners and reach the downstream inviscid conditions quickly. The downstream influence is less than one boundary-layer thickness for all three test cases. This agrees with the discussion of Narasimha and Sreenivasan.[7] The extent of the interaction region of a lower supersonic expansive flow is relatively insensitive to the corner flow deflection. Thus the downstream influence scaling proposed by Lu and Chung[6] would be valid for the expansive flow at higher Mach number. The lower supersonic expansive flow can be treated as inviscid.

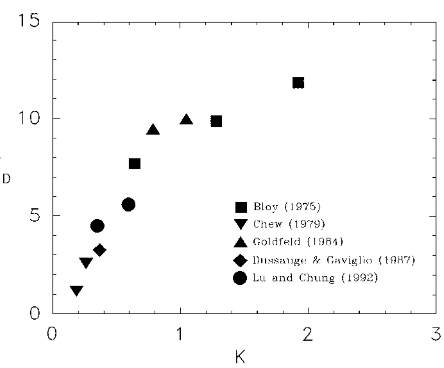

Fig. 1 Downstream influence.[6]

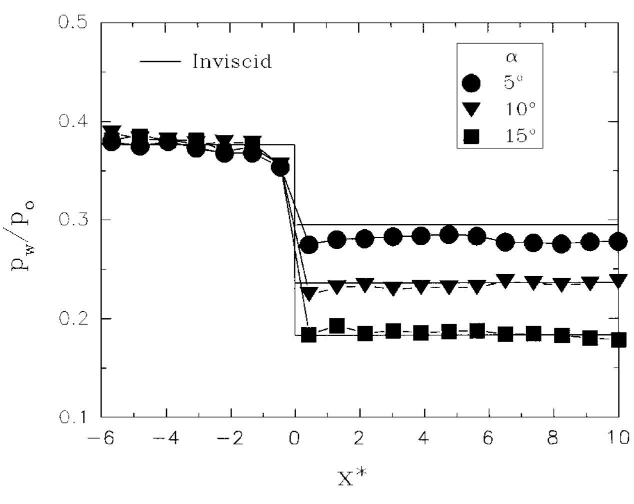

Fig. 2 Surface pressure distributions.

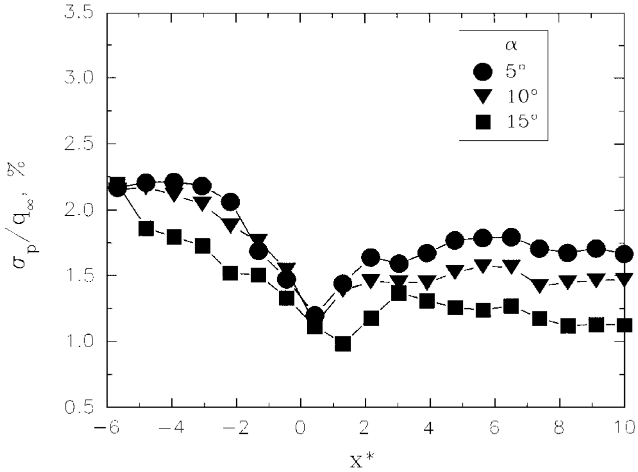

Fig. 3 Surface pressure fluctuations.

Further, the distributions of surface pressure fluctuations are shown in Fig. 3. The pressure fluctuations σ_p are normalized by upstream dynamic pressure q_∞. The distributions indicate that the intensity of pressure fluctuations decreases and reaches the minimum downstream of expansion corners. The final equilibrium pressure fluctuation levels are obtained within three- to five-boundary-layer thicknesses, which indicates a larger interaction region than that obtained from the mean surface pressure distributions. Thus the interaction region scaling of the expansive flows may be further refined based on the distribution of surface pressure fluctuation.

Conclusions

Experiments were carried out to study the effects of expansion corners on a turbulent boundary layer. At this lower supersonic Mach number ($M_\infty = 1.280$), the extent of the interaction region is considered neligible based on mean surface pressure distributions. The downstream influence for a lower supersonic, expansion-corner flow is insensitive to the corner deflection angle.

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References

[1]Adamson, T. C., "Effect of Transport Properties on Supersonic Expansion Around a Corner," *Physics of Fluids*, Vol. 10, No. 5, 1967, pp. 953–962.

[2]Bloy, A. W., "The Expansion of a Hypersonic Turbulent Layer at a Sharp Corner," *Journal of Fluid Mechanics*, Vol. 67, Pt. 4, 1975, pp. 647–655.

[3]Chew, Y. T., and Squire, L. C., "The Boundary Layer Development Downstream of a Shock Interaction at an Expansion Corner," Aeronautical Research Council, Repts. and Memoranda 3839, Churchill College, Cambridge, England, UK, Aug. 1979.

[4]Dussauge, J. P., and Gaviglio, J., "The Rapid Expansion of a Supersonic Turbulent Flow: Role of Bulk Dilatation," *Journal of Fluid Mechanics*, Vol. 174, 1987, pp. 81–112.

[5]Goldfeld, M. A., "On the Reverse Transition of Compressible Turbulent Boundary Layer in a Transverse Flow Around a Convex Corner Configuration," *Proceedings of the IUTAM Symposium on Laminar-Turbulent Transition*, edited by V. V. Kozlov, Springer–Verlag, Berlin, Germany, 1985, pp. 515–520.

[6]Lu, F. K., and Chung, K. M., "Downstream Influence Scaling of Turbulent Flow Past Expansion Corners," *AIAA Journal*, Vol. 30, No. 12, 1992, pp. 2976, 2977.

[7]Narasimha, R., and Sreenivasan, K. R., "Relaminarization of Fluid Flows," *Advances in Applied Mechanics*, Vol. 19, 1979, pp. 221–309.

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